Cyclic MUSIC algorithm for DOA estimation of wideband coherent signals in frequency domain

Ivan P. Pokrajac, Desimir Vucic, Miljko Eric and M. L. Dukic.

***Abstract* — The multipath propagation due to various reflections is often encountered in wireless communication systems. In this paper is investigated possibility to use the Cyclic MUSIC algorithm in frequency domain for direction of arrival (DOA) estimation of wideband coherent signals without using the spatial smoothing technique. The performances of the proposed algorithms are verified through numerical examples and some results are shown in this paper.**

***Key words* — cyclostationarity, direction of arrival, MUSIC, wideband coherent signal.**

I. Introduction

MANY signal selective algorithms for DOA estimation have been developed to overcome some limitations of existing algorithms, such as the possibility to automatically classify signals as desired or undesired. The first signal selective algorithm based on cyclostationarity properties of signal is proposed by Gardner [1], and named Cyclic MUSIC. Since then many algorithms for DOA estimation have been developed to improve performance of Cyclic MUSIC, such as those in [2] - [7]. Some of them are designed only for narrowband cyclostationary signals, such as [3] and [6], and other for wideband cyclostationary signals. However, the most serious limitation of algorithms presented in [2] - [4] is reflected in the fact that the cyclic correlation is evaluated in time domain using one or more time lag parameters τ. To solve this problem, the algorithms for DOA estimation in frequency domain based on cyclostationarity properties of signal are proposed in [5] and [6]. The spectral cyclic correlation matrix (SCCM) is evaluated in the proposed algorithms for DOA estimation of the wideband and the narrowband signals, instead of the cyclic correlation matrix (CCM). Algorithms for DOA estimation of wideband signal based on cyclostationarity in frequency domain but using spectral conjugate cyclic correlation matrix (SCCCM) or extended spectral cyclic correlation matrix (ESCCM) are proposed in [7].

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There are a lot of papers in the literature which deal with the problem of DOA estimation of wideband signals, but only a few papers about DOA estimation of wideband coherent signals by cyclostationary properties [8]. In these presented algorithms CCM is evaluated in time domain. To our knowledge there is no any work about DOA estimation of wideband coherent signals using cyclostationarity properties in frequency domain without the spatial smoothing (SS) technique.

As a signal radiated from a source arrives at antenna array from different directions, under multipath propagation, the signals in the various paths are correlated and the cyclic spectral correlation matrix will become singular. In this paper, we investigate possibility to use Cyclic MUSIC algorithm in frequency domain [7] to estimate DOA of wideband coherent signal without the spatial smoothing technique like it is presented in [9]. Possibility to estimate DOAs of wideband coherent signals choosing the optimal cyclic resolution for evaluating SCCM is shown in this work. Evaluating SCCM in frequency domain under multipath environment without the spatial smoothing technique is possible to overcome some disadvantages which appear using SS technique such as: a smaller effective array aperture, reducing the angle resolution and decreasing the total number of superposed signals which can be detected.

II. Data model

Consider an uniform antenna array, composed of L omni-directional elements, which receive K uncorrelated wideband signals $u_k(t), k=1,\ldots,K$. Suppose that under multipath propagation signal from the kth source arrive at the antenna array from R_k different directions and signals from different paths are correlated. The kth signal for the pth path can be expressed in time domain by:

$$u_k^{(p)}(t)=\beta_k^{(p)}\cdot u_k\left(t-\tau_k^{(p)}\right)\exp\left(j2\pi f_{Dk}^{(p)}\right), k=1,\ldots,K. \quad (1)$$

where $\beta_k^{(p)}$ is the multiplath coefficient which represents the compolex attenuation of the kth signal for the pth path. $\tau_k^{(p)}$ is time delay with respect to the direct path ($p=1$), and $f_{Dk}^{(p)}$ is Doppler shift with respect to the central frequency of the kth signal.

Signal $u_k^{(p)}(t)$ can be rewritten as:

$$u_k^{(p)}(t)=\beta_k^{(p)}s_k\left(t-\tau_k^{(p)}\right)\cdot\exp\left(j2\pi f_{Ck}\left(t-\tau_k^{(p)}\right)\right)\exp\left(j2\pi f_{Dk}^{(p)}t\right) \quad (2)$$

where f_{Ck} is central frequency of the kth signal and $s_k(t)$ is kth complex signal in base band. At the receiver front end, the received continuous time signal $u_k^{(p)}(t)$ is down converted to baseband by frequency f_C :

$$f_k^{(p)}(t)=\beta_k^{(p)}s_k\left(t-\tau_k^{(p)}\right)\cdot\exp\left(j2\pi f_{Ck}\left(t-\tau_k^{(p)}\right)\right)\\ \exp\left(j2\pi f_{Dk}^{(p)}t\right)\exp\left(-j2\pi f_C t\right) \tag{3}$$

Relation in (3) can be rewritten as:

$$f_k^{(p)}(t)=\beta_k^{(p)}s_k\left(t-\tau_k^{(p)}\right)\cdot\exp\left(j2\pi\left(f_{Ck}-f_C\right)t\right)\\ \exp\left(j2\pi f_{Dk}^{(p)}t\right)\exp\left(-j2\pi f_{Ck}\tau_k^{(p)}\right) \tag{4}$$

where $f_k^{(p)}(t)$ is the equivalent complex signal down converted to baseband. f_h denotes the hth frequency component, $h\in 1,H$, where H is the total number of spectral components for the selected frequency sub-band. In frequency domain relation in (4) can be express as:

$$F_k^{(p)}(f_h)=\beta_k^{(p)}S_k\left(f_h-f_{Ck}-f_{Dk}^{(p)}\right)\cdot\\ \exp\left(-j2\pi f_{Ck}\tau_k^{(p)}\right)\exp\left(-j2\pi f_h\tau_k^{(p)}\right) \tag{5}$$

Under multipath environment the overall number of impinging signals is greater from K and it is $K_1=\sum_{k=1}^{K}R_k$, where R_k is the number of all paths from kth source including and direct path. The mathematical model of received wideband coherent signal at antenna array can be expressed using model of received wideband uncorrelated signals as is shown in [10].

It is supposed that the wave fronts of K_1 signal impinge on the antenna array from directions $\Theta_1,\Theta_2,\ldots,\Theta_{K_1}$, where Θ_k represents azimuth θ_k and elevation φ_k of the kth signal. If we consider that $K_\alpha=\sum_{m=1}^{M}R_m$ from K_1 signals are contained in vector $\mathbf{f}(t)$, where is M the number of uncorrelated signals which exhibit cyclic features at cycle frequency α (with conditions $K_m\le K$), then K_1-K_α signals, which exhibit cyclic features at cycle frequencies different from α and any noise are lumped into vector $\mathbf{i}(t)$. Based on this assumption, the wideband received signal can be expressed in frequency domain as [10]:

$$\mathbf{X}(f_h)=\mathbf{A}(\Theta,f_C,f_h)\mathbf{F}(f_h)+\mathbf{I}(f_h) \tag{6}$$

Where:

$\mathbf{X}(f_h)=\left[X_1(f_h)\ X_2(f_h)\ \ldots\ X_L(f_h)\right]^T$ is a vector of frequency components of the wideband signals received in the observed interval T,

$\mathbf{F}(f_h)=\left[F_1^{(1)}(f_h)\ \cdots\ F_1^{(R_1)}(f_h)\ \cdots\ F_M^{(1)}(f_h)\ \cdots\ F_M^{(R_M)}(f_h)\right]$ is a vector of frequency components of K_α signals down converted to base band in the same observed interval,

$\mathbf{I}(f_h)=\left[I_1(f_h)\ I_2(f_h)\ \ldots\ I_L(f_h)\right]^T$ is a vector which contains frequency components of the interfering signals and white Gaussian noise.

Matrix $\mathbf{A}(\Theta,f_h)=\left[\mathbf{a}(\Theta_1,f_C,f_h),\ldots,\mathbf{a}(\Theta_{K_\alpha},f_C,f_h)\right]^T$ contains the steering vectors of the signals of interest (SOIs).

III. DOAs ESTIMATION OF WIDEBAND COHERENT SIGNALS

In order to estimate DOA at selected cyclic frequency it is necessary to evaluate the spectral cyclic correlation matrix $\mathbf{R}_{XX}^{\alpha}(f_h)=\mathbf{A}\left(\Theta,f_h+\frac{\alpha}{2}\right)\cdot\mathbf{R}_{FF}^{\alpha}(f_h)\cdot\mathbf{A}\left(\Theta,f_h-\frac{\alpha}{2}\right)^H$ under conditions that contribution to the SCCM from K_1-K_α signals and from any noise converges to zero as integration time tends to infinitive [9]. It is supposed that one signal arrive at antenna array from two different paths. In that case, signals can be expressed in frequency domain by (5), where $k=1$ and $p=1,2$. The SCCM $\mathbf{R}_{FF}^{\alpha}(f_h)$ can be expressed as [6]:

$$\mathbf{R}_{FF}^{\alpha}(f_h)=\left\langle\mathbf{F}_1\left(f_h+\frac{\alpha}{2}\right)\cdot\mathbf{F}_1^{*}\left(f_h-\frac{\alpha}{2}\right)\right\rangle_{\Delta f}=\begin{bmatrix}R_{FF_{11}} & R_{FF_{12}}\\ R_{FF_{21}} & R_{FF_{22}}\end{bmatrix} \tag{7}$$

where is $\mathbf{F}_1\left(f_h+\frac{\alpha}{2}\right)=\left[F_1^{(1)}(f_h+\alpha/2)\quad F_1^{(2)}(f_h+\alpha/2)\right]^T$.

Based on (5) and (7) elements of $\mathbf{R}_{FF}^{\alpha}(f_h)$ can express as:

$$R_{FF_{11}}=\beta_1^{(1)}\beta_1^{(1)*}R_{SS}^{\alpha}\left(f_h-f_{D1}^{(1)}\right)\cdot\exp\left(-j2\pi\alpha\tau_1^{(1)}\right) \tag{8}$$

and

$$R_{FF_{12}}=\beta_1^{(1)}\beta_1^{(2)*}S_1\left(f_h+\frac{\alpha}{2}-f_{D1}^{(1)}\right)S_1\left(f_h-\frac{\alpha}{2}-f_{D1}^{(2)}\right)^{*}\\ \exp\left(-j2\pi f_{C_1}\Delta\tau_1^{(1,2)}\right)\cdot\exp\left(-j2\pi\left(f_h\Delta\tau_1^{(1,2)}+\frac{\alpha}{2}\Sigma\tau_1^{(1,2)}\right)\right) \tag{9}$$

where is $\Delta\tau_1^{(1,2)}=\tau_1^{(1)}-\tau_1^{(2)}$ and $\Sigma\tau_1^{(1,2)}=\tau_1^{(1)}+\tau_1^{(2)}$.

In [10] is shown that is possible to use MUSIC algorithm for DOA estimation of coherent signals under condition that $\left|r_{12}\right|=\left|R_{FF_{12}}\middle/\sqrt{\left|R_{FF_{11}}\right|^2\left|R_{FF_{22}}\right|^2}\right|<1$, where is r_{12} the normalized correlation coefficient. In this case the rank of $\mathbf{R}_{FF}^{\alpha}(f_h)$ is full and signals are not coherent. In case that $\Delta\tau_1^{(1,2)}=0$ and $f_{D1}^{(1)}=f_{D1}^{(2)}$ the normalized correlation coefficient is $\left|r_{12}\right|=1$ and the rank of $\mathbf{R}_{FF}^{\alpha}(f_h)$ is one and signals are coherent.

In this work is analyzed how the normalized correlation coefficient r_{12} dependence on time delay or Doppler shift. In the first case it is assumed that only there is time delay between signals. In case that there is no Doppler shift elements $R_{FF_{11}}$ and $R_{FF_{12}}$ can be expressed as:

$$R_{FF_{11}}=\beta_1^{(1)}\beta_1^{(1)*}R_{SS}^{\alpha}(f_h)\cdot\exp\left(-j2\pi\alpha\tau_1^{(1)}\right) \tag{10}$$

and

$$R_{FF_{12}}=\beta_1^{(1)}\beta_1^{(2)*}R_{SS}^{\alpha}(f_h)\exp\left(-j2\pi f_{C_1}\Delta\tau_1^{(1,2)}\right)\cdot\\ \exp\left(-j2\pi\left(f_h\Delta\tau_1^{(1,2)}+\frac{\alpha}{2}\Sigma\tau_1^{(1,2)}\right)\right) \tag{11}$$

Based on (10) and (11) the normalized correlation coefficient r_{12} can be expressed as:

$$r_{12}=\frac{\beta_1^{(1)}\beta_1^{(2)*}R_{SS}^{\alpha}(f_h)\exp\left(-j2\pi f_{C_1}\Delta\tau_1^{(1,2)}\right)\cdot\exp\left(-j2\pi\left(f_h\Delta\tau_1^{(1,2)}+\frac{\alpha}{2}\Sigma\tau_1^{(1,2)}\right)\right)}{\sqrt{\left(\beta_1^{(1)}\beta_1^{(2)}\right)^2\cdot R_{SS}^{\alpha}(f_h)^2}} \tag{12}$$

Based on (12) can be concluded that only $\Delta\tau_1^{(1,2)}$ and $\Sigma\tau_1^{(1,2)}$ have effect on the magnitude of normalized correlation coefficient $|r_{12}|$.

The DOAs estimation of wideband coherent signals is likely in case when is $|r_{12}|=0$. Based on [10] the normalized correlation coefficient of coherent signals inside one spectral component can be expressed as:

$$r_{12}=\frac{\sin\left(2\pi\left(\frac{\Delta f_{BW}}{H}\Delta\tau_1^{(1,2)}+\frac{\Delta f_{BW}}{2H}\Sigma\tau_1^{(1,2)}\right)\right)}{2\pi\left(\frac{\Delta f_{BW}}{H}\Delta\tau_1^{(1,2)}+\frac{\Delta f_{BW}}{2H}\Sigma\tau_1^{(1,2)}\right)} \quad (13)$$

Based on (13) can be concluded that is $|r_{12}|=0$ when is satisfied following condition:

$$\frac{\Delta f_{BW}}{H}\left(\Delta\tau_1^{(1,2)}+\frac{\Sigma\tau_1^{(1,2)}}{2}\right)=0.5 \quad (14)$$

We can conclude that increasing cyclic resolution (decreasing the number of spectral components H for estimation of SCCM) is possible to decrease correlation between coherent for the same time delay.

In second case it is assumed that there is only Doppler shift between signals which arrive at antenna array from various paths. In this case, elements $R_{FF_{11}}$ and $R_{FF_{12}}$ can be expressed as:

$$R_{FF_{11}}=\beta_1^{(1)}\beta_1^{(1)*}R_{SS}^{\alpha}\left(f_h-f_{D1}^{(1)}\right) \quad (15)$$

$$R_{FF_{12}}=\beta_1^{(1)}\beta_1^{(2)*}R_{SS}^{\alpha_1}\left(f_h-\frac{\Sigma f_{D1}^{(1,2)}}{2}\right) \quad (16)$$

where is $\alpha_1=\alpha+f_{D1}^{(2)}-f_{D1}^{(1)}=\alpha+\Delta f_{D1}^{(1,2)}$ and $\Sigma f_{D1}^{(1,2)}=f_{D1}^{(1)}+f_{D1}^{(2)}$.

Based on (16) can be concluded that $R_{FF_{12}}$ exhibits cyclostationarity properties at cyclic frequencies $\alpha\pm\left|\Delta f_{D1}^{(1,2)}\right|$ different from $R_{FF_{11}}$. In this case the problem of resolving DOAs of wideband coherent signals is the same as the problem of estimating DOAs at two close cyclic frequencies. In order to estimate DOAs at two close cyclic frequencies it is necessary to decrease cyclic resolution. Choosing optimal cyclic resolution it is possible to separate coherent signals by Cyclic MUSIC algorithm in frequency domain when there is Doppler shift. However, increasing the number of spectral components H in order to estimate DOAs of coherent signals has opposite effect on DOAs estimation in case of time delay between coherent signals.

IV. Simulation results

The performances of the Cyclic MUSIC algorithm in case of DOAs estimation of wideband coherent signals are verified through the following examples. Signals are received by a linear, uniformly spaced array with nine antennas, spaced by a half wavelength of frequency $f_A=30$ MHz. The central frequency of the selected frequency band $f_{BW}=19.2$ MHz is $f_C=20$ MHz.

In the first example is investigated possibility to separate two coherent signals, when there is only time delay between them. In this example two wideband BPSKs signals arrive from azimuth of -40º and 20º at antenna array with the bit rate of 6.4 Mb/s and 4.8 Mb/s. Reflected signals arrive from azimuth of -20 º and 40º for the first and the second signal, respectively. The signal to noise ratio (SNR) is 10 dB for the first signal and 15 dB for the second signal. The central frequencies of superposed signals are same and correspond to the central frequency f_C of selected frequency band.

The multipath coefficients which represents the complex attenuations are $\beta_1^{(1)}=1$ and $\beta_1^{(2)}=0.32-0.7j$ for the first signal and $\beta_2^{(1)}=1$ and $\beta_2^{(2)}=0.2+0.7j$ for the second signal. It is supposed that reflected signals are time delayed for one bit period. Time delay between signals is $\Delta\tau_1^{(1,2)}=\Sigma\tau_1^{(1,2)}=0.15\,\mu s$ for the first signal and $\Delta\tau_1^{(1,2)}=\Sigma\tau_1^{(1,2)}=0.21\,\mu s$ for the second signal.

DOAs estimation is performed under assumption that the cyclic frequency at which signal exhibits cycle properties is known to the Cyclic MUSIC algorithm. The results of DOAs estimation of wideband coherent signals at cycle frequency α=4.8 MHz is shown in Fig.1 and Fig.2. The spectral cyclic correlation matrix is evaluated at $H=256$ points in observed time interval $\Delta T=5\,ms$ in the first case (Fig.1), and in the second case $H=1024$ points in the same time interval (Fig.2).

Based on results shown in Fig.1 can be concluded that is possible to resolve DOAs of wideband coherent signals by the Cyclic MUSIC algorithm in frequency domain decreasing the total number of spectral components H (increasing cyclic resolution in the same observed time interval).

In the second example it is supposed that there is only Doppler shift between coherent signals. It is used the same scenario like in the first examples. The Doppler shifts for signals are $f_{D1}^{(1)}=0$ kHz, $f_{D1}^{(21)}=50$ kHz, $f_{D1}^{(21)}=0$ kHz and $f_{D1}^{(2)}=12.5$ kHz.

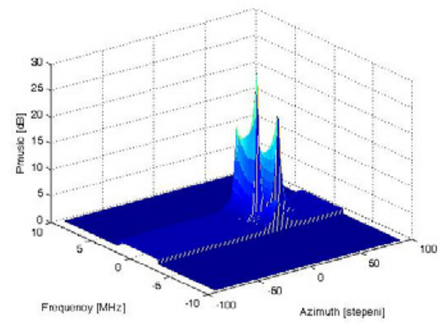

Fig. 1. DOA estimation of wideband coherent BPSKs signals (direct path from 20º and reflected path from 40º) at cyclic frequency α=4.8 MHz (H=256 and $\Delta T=5$ ms).

The results of DOAs estimation of wideband coherent signals at cycle frequency α=4.8125 MHz is shown in Fig.3. The spectral cyclic correlation matrix is estimated at $H=8192$ points in observed time interval $\Delta T=20\,ms$. Based on results shown in Fig.3 can be concluded that is detected only one signal which exhibits cyclic properties at selected cycle frequency, and that its DOA is estimated correctly. Similar results are obtained for DOA estimation at cycle frequency α=6.45 MHz. In this case the spectral cyclic correlation matrix is estimated at $H = 4096$ points. Based on results shown in Fig.4 can be concluded that only signal which arrive at antenna array from reflected path exhibits cycle properties at selected cycle frequency.

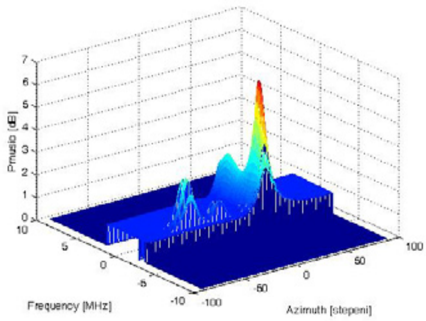

Fig. 2. DOA estimation of wideband coherent BPSK signal (direct path from 20º and reflected path from 40º) at cyclic frequency α=4.8 MHz (H=1024 and $\Delta T = 5$ ms).

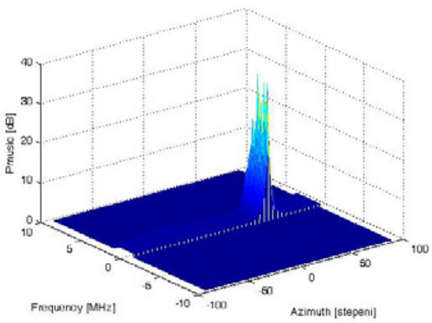

Fig. 3. DOA estimation of wideband coherent BPSKs signals (reflected path from 40º) at cyclic frequency α=4.8125 MHz (H=8192 and $\Delta T = 20$ ms).

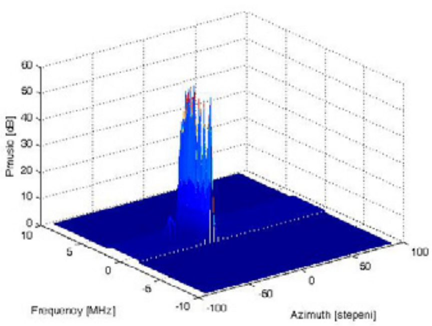

Fig. 4. DOA estimation of wideband coherent BPSK signal (reflected path from -20º) at cyclic frequency α=6.45 MHz (H=4096 and $\Delta T = 20$ ms).

DOAs estimation at cycle frequency α=6.4 MHz is shown in Fig.5. Based on shown results can be concluded that is possible to separate DOAs of wideband coherent signals using proposed algorithms in frequency domain increasing the total number of spectral components H, or decreasing spectral resolution.

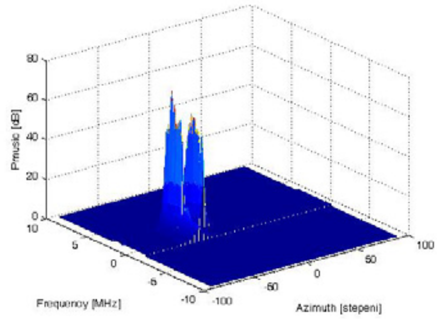

Fig. 5. DOA estimation of wideband coherent BPSKs signals (direct path from -20º and reflected path from -40º) at cyclic frequency α=6.4 MHz (H=4096 and $\Delta T = 20$ ms).

V. Conclusion

Recently many algorithms for DOA estimation of narrowband coherent signals by cyclostationarity using spatial smoothing method have been proposed. Unfortunately, these algorithms have same disadvantages. In this paper, we analyzed possibility to use the Cyclic MUSIC algorithm in frequency domain for DOA estimation of wideband coherent signals without spatial smoothing technique. It is clarified that if it is chosen optimal spectral or cyclic resolution in order to decrease the correlation coefficient between coherent signals it is possible to resolve theirs DOAs.

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